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ABSTRACT

This is volume three of a three-volume SMSG junior high school mathematics text. This volume includes the units concerned with applications of mathematics. Unit topics include: (1) what is mathematics and why you need to know it; (2) mathematics at work in science; (3) Uncle Sam as a statistician; and (4) chance. (MP)

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JUNIOR HIGH SCHOOL MATHEMATICS UNITS

VOLUME III, APPLICATIONS

Prepared by the SCHOOL MATHEMATICS STUDY GROUP
Under a grant from the NATIONAL SCIENCE FOUNDATION

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These units were prepared in the summer of 1958 at Yale University by a School Mathematics Study Group writing team. The units were taught in a number of classes during the academic year 1958-59. No major editing of them has been attempted, but typographical and other errors have been corrected.

This volume includes the units concerned with applications of mathematics. These are units I, XI, XII, XIII in the numbering system originally used.

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UNIT I

I. WHAT IS MATHEMATICS AND WHY YOU NEED TO KNOW IT

> "Once, on a train, I fell into conversation with the man next to me. He asked me what kind of work I do. I told him I was a mathematician. He exclaimed, 'You are! Don't you get tired of adding columns of figures all day long?' I had to explain to him that adding numbers can be done by a machine. My job is mainly logical reasoning."

What is this mathematics that so many people are talking about these days? Is it counting and computing? Is it measuring and drawing? Is it a language that uses symbols, like a code? No, mathematics is not just any one of these; it is much more than all of these. Mathematics is a way of thinking, a way of reasoning. It is a science of reasoning called deductive reasoning.

Let us take an example of reasoning. Suppose there are thirty pupils in your classroom. How can you prove that there are at least two pupils who have birthdays during the same month? To prove this statement you don't need to know the birthdays of the pupils in your room. Instead, you reason like this. Imagine 12 boxes, one for each month of the year. Imagine also that your teacher wr es each pupil's name on a slip of paper, then puts each slip into the proper box according to the pupil's birthday. If no box has more than one name, then there could not be more than 12 names. Since there are 30 names then at least one box must contain more than one name.

This is what a mathematician does. He proves statements using logical reasoning. Notice exactly what you proved. You proved that if there are 30 pupils in your room then there are at least two pupils who were born the same month. It is not the mathematician's job to find out whether there are 30 pupils in your room. This can be found out by observation; the teacher can count the pupils. The mathematician's business is the "if -- then" statement. By reasoning he tries to prove that if something is true, then something else must be true.

In arithmetic you have learned how to prove simple statements about numbers. You can tell, without seeing the bags of peanuts and without counting, that if you have 7 bags with 5 peanuts in each bag, then you have 35 peanuts altogether. You can solve more difficult problems by reasoning instead of calculation.

Exercise.

1. What is the least number of students that could be enrolled in a school so that one could be sure there are at least two students with the same birthdate?

2. What is the largest number of students that could be enrolled in a school so that one could be sure that no two students have the same birthdate? 364? 365? Less than 364?

3. If there are 12 movie houses in a certain town, how many people have to go to the show before you can be sure that there will be at least two people in one show?

4. If there are five movie houses in a town what is the smallest number of people that would have to go to the movies before you can be sure that at least two people will see the same show?

5. What is the largest number of people that could go to the movie houses so that you could be sure that no two people see the same show?

6. If 8 candy bars are to be divided among 5 boys, how many boys can receive three candy bars if each boy is to receive at least one bar?

7. A 200 pound man and his two sons each weighing 100 pounds want to cross a river. If they have only one boat which can safely carry only 200 pounds, how can they cross the river?

8. A farmer wants to take a goose, a fox, and a bag of corn across a river. If left alone the fox will eat the goose or the goose will eat the corn. If he has only one boat large enough to carry him and one of the others, how does he cross the river?

BRAINBUSTER

9. Three cannibals and three missionaries want to cross a river. They must share a boat which is large enough to

carry only two people. At no time may the cannibals outnumber the missionaries, but the missionaries may outnumber the cannibals. If they work together, how can they all cross the river?

BRAINBUSTER

10. Eight marbles all have the same size, color, and shape. Only one marble differs in weight. Using a balance scale, can you find the heavy marble if you make only two weighings?

Another way scientists solve problems is to make experiments and observation. This method is called the experimental method. When you use this method you experiment like a scientist in a laboratory. In many problems such an experimental method works. Can you think of some problems that have been solved in this way? A good example is the story of how Edison invented the electric lamp. Try to find out how Edison developed some of his inventions.

Of course the part of mathematics which you know best right now is arithmetic. Even in arithmetic you can obtain results by deductive reasoning which would be hard to obtain by ordinary calculation. When Karl Friedrich Gauss was about 10 years old, his teacher wanted to keep the class quiet for a while. He told the children to add all the numbers from 1 to 100. In about two minutes Gauss was up to mischief again. The teacher asked him why he wasn't working on the problem. He replied "I've done it already!" "Impossible!" exclaimed the teacher. "It's easy," answered Gauss. "I wrote

$$1 + 2 + 3 \ldots + 100$$ and then I reversed the numbers

$$\underline{100 + 99 + 98 \ldots + 1}$$ and then I added each pair of numbers

$$101 + 101 + 101 \ldots + 101$$

When I add, I get a hundred 101's, which gives me $100 \cdot 101 = 10{,}100$. But I used each number twice, so I divided my answer by 2." The answer is $\frac{10{,}100}{2} = 5{,}050$

Who was Karl Gauss? Where did he live? When did he live? What did he do for a living?

Exercise.

1. Add all the numbers from 1 to 5 (1 + 2 + 3 + 4 + 5) using Gauss' method. Try to discover the "short" method used by Gauss.

2. Add 0 + 2 + 4 + 6 + 8. Can you do this mentally? How many digits are there in our problem? What is the sum of the first and last digit? 8 + 0 = ? Find the product 5 · 8. Take half of this answer. 1/2 of 40 = 20. Is this the correct sum?

3. Add 1 + 2 + 3 using Gauss' short method.

4. Add 2 + 4 + 6. Does the short method work?

5. Add the numbers 0 + 1 + 2 + 3 ... + 8. Does the short method work?

   (three dots written . . . mean "and so on." a dot is used for each word.)

6. Add the odd numbers from 1 to 15. 1 + 3 + 5 ... + 15.

7. Add the even numbers up to 20. Start with 0 + 2 + 4 + 6 ... + 20.

8. Add the numbers 1 + 2 + 3 ... 25.

9. Add the odd numbers 7 + 9 + 11 ... + 15.

10. Add the even numbers from 22 to 30. 22 + 24 + 26 ... + 30.

11. Add all the numbers from 0 to 50. 0 + 1 + 2 + 3 + 4 ... + 50.

BRAINBUSTER

12. Add all the numbers from 1 to 200. 1 + 2 + 3 + 4 . . . + 200. Then add all the numbers from 0 to 200. 0 + 1 + 2 + 3 + 4 ... + 200. Are the answers for these two problems the same? Why? Will this method work if we start with 0? If we start with 1? Will this method work for a number series if we start at some place other than 0 or 1? Can you determine why?

A mathematician reasons about all sorts of things. He thinks about numbers and shapes. He tries to figure out the lifting force on an airplane wing. He tries to predict how many automobiles will be sold next year. Everywhere you look you find things to wonder about. If you are full of curiosity you

will enjoy being the detective in a life-long mystery story.

The physicist and the engineer make experiments to find out the laws of the flow of air. Mathematicians start from the laws which engineers have discovered by observation. Mathematicians try to find out, by reasoning, how air will flow around a wing of a certain shape if the airplane is moving with a certain speed. They try to predict the lifting force on the wing. The engineer then makes a model airplane and tests it in a wind tunnel. If the mathematician's prediction works, then the engineer may use it to design a new airplane. If the mathematician's prediction does not work he will have to change the "if" part of his reasoning and try again. Nowadays, it takes 100,000 hours of paper work before an engineer can begin the planning work for designing a new airplane.

When Walt Disney considered setting up Disneyland, he had many questions. He wanted to know how big it should be, where it should be, what admission price should be charged, what special facilities he should have for the Fourth of July and other holidays. He didn't want to start building Disneyland at a cost of $17,000,000 until he knew the answers to these questions. No one likes to take a chance on losing that much money. Disney went to the Stanford Research Institute where there is a group of mathematically trained people who are specialists on applying mathematical reasoning to business problems. They gave him the answers he needed to start building.

One topic of interest to mathematicians is probability or chance. Probability is important in many games such as a game with cards. Probability is also important in business, in government, and in science. For example, a mathematician uses probability to predict how many people will be killed in automobile accidents. Government experts predict how much food will be produced by farmers even before the grain is

planted. A scientist will predict how many radioactive molecules will fall on your yard tonight. The weatherman uses probability to forecast tomorrow's weather. By using probability with agricultural experiments scientists obtained hybrid corn and rust resistant wheat.

Most mathematicians are not mainly interested in airplanes or farming or business or government. They are mostly interested in the reasoning which is used in solving the problems which arise in these fields. They feel about their work more as a painter or a composer or a poet. Mathematicians find beauty and fascination in the new ideas they discover. You may say that mathematicians work for the fun of it. But the rest of the world pays them to enjoy themselves because they find the results useful. Sometimes a mathematician works on a practical problem. Often his work turns out to be useful only many years after his death.

Exercise.

To see how probability works, imagine that you toss two pennies. There are 4 possible ways that the coins can come up:

	First Penny			
Second Penny	H	H	T	T
	H	T	H	T

We are using H to represent heads and T to represent tails. HH describes the event of 2 coins coming up heads. We say then that the possibility of tossing 2 heads with 2 coins is 1 out of 4 or 1/4. We cannot predict what will happen in any one toss, but we can predict that if the two coins are tossed 100,000 times then it is likely that two heads will come up about 1/4 of the time.

1. What is the probability of drawing the ace of spades from a deck of cards?

2. What is the probability of drawing an ace from a full deck of cards?

3. What is the probability of throwing a die (one of a pair of dice) and having two dots come up?

BRAINBUSTERS

4. What is the probability of throwing a pair of ones (one dot on each die) with one pair of dice?

5. What is the probability of having 3 heads come up if 3 pennies are tossed?

Why you need to know mathematics

You are living in a world which is changing very rapidly. Scientists are making medical discoveries so that people live longer. Scientists and engineers are finding new things to make and do each year.

If you want to know how much our world has changed, ask your parents and grandparents what life was like when they were in school. How did their way of life compare with ours? You will probably live through more changes and face more new kinds of problems than either your parents or grandparents.

No one knows what these changes will be, or what things you will need to know when you grow up. In fact, you will need to learn very many things which haven't as yet been discovered. Therefore, you need to know, more than anything else, how to get new knowledge. Since any job you get will change very rapidly, your employer will not be as interested in what you know as in how able you are in learning more and in understanding new ideas.

There are two main ways of learning about the world around you. One is the method of observing and experimenting. This is called the inductive method. This is the main method used by the scientists. The other is the method of logical reasoning. This is the deductive method.

The scientist, engineer, and mathematician use both of these methods. You will understand the world around you better if you know how to use these methods.

You should also learn as much as possible about mathematics in other jobs because an ever-growing number of jobs demand more and more mathematical training. As an example take the departments of the government.

Many departments of the government use mathematics in their daily work. The government and private industry employ many mathematicians and mathematically trained people in physics, chemistry, biology, engineering, economics and psychology. There are now about 7,000 professional mathematicians in the United States. The computing machines now on order would require more than twice that number of mathematicians. Even if you don't use mathematics yourself in whatever job you get, and there will be fewer jobs like that as time goes on, you will also have to know how mathematics is used in every phase of modern life in order to be a good citizen. For example, most of the bills before Congress today involve mathematics or science in some way.

Finally when you grow up, you will have more leisure than people have had before. You can choose between a dull, empty life or an exciting, full life. When you learn a way of creating beauty yourself and appreciating the creation of others, you will enjoy an exciting life. By learning mathematics, what it is and how it works, you will be prepared for a more successful and creative life.

UNIT XI

THE SCIENTIFIC SEESAW OR MATHEMATICS AT WORK IN SCIENCE

Have you ever played on a seesaw?

If your weight is 100 pounds and your partner on the other side of the seesaw weighs 85 pounds, where does he have to sit so that the two sides will just balance? Will he be closer to the center or farther away from it than you? How far?

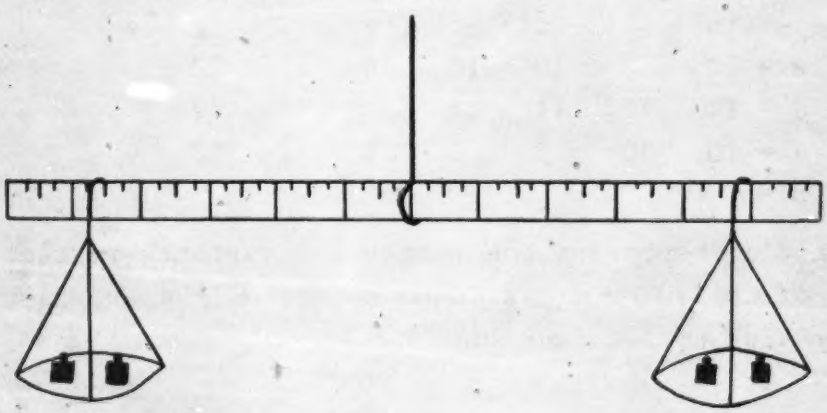
The seesaw is a form of simple machine that is used a great deal by scientists. It belongs to the family of machines called "levers." Scientists have investigated how it works and how to balance weights on it, and they have expressed their findings in a mathematical formula. Levers are used in very many ways.

Today you will be the scientist. You will set up the equipment, make the observations, discover the rule or law, and state it in mathematical form.

The following experiment is one example of how a scientist makes certain observations in the laboratory, studies them mathematically and draws conclusions from them, states the conclusions by means of a mathematical formula, makes predictions, and then goes back to the laboratory to test whether the formula works in any similar situation.

EXPERIMENT

To begin to study this scientific seesaw, your equipment will look something like this:



Materials:

A meter stick or yardstick
String and 2 pans or bags to hold the weights
A set of metric weights. (If a set of weights is not available, a batch of pennies can be used.)

Procedure

1. Balance the bar by suspending it from a string tied at a point near the middle. This point is called the fulcrum.

2. Place one weight on one side of the fulcrum and another weight on the other side, and try to make the lever balance. Try some other weights and make them balance. Do you find that you have to change the distance according to the size of the weights?

Note: Scientists do not usually make their discoveries with haphazard trials as you have just been doing, but only after they have set up a very carefully planned experiment.

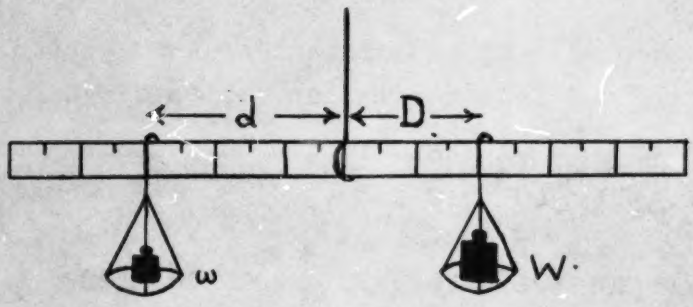
Let us set up a plan and see what we can discover about the lever with the aid of mathematics.

3.(a). Hang a weight of 10 grams (or 10 pennies if you do not have metric weights) at a distance of 12 centimeters from the fulcrum, and balance it with a 10 gram weight on the other side. Observe the distance of this second weight when the lever is in balance and record the distance in a table, column (a), similar to the one below.

TABLE I

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
w =	10	10	10	10	10	10	10
d =	12	12	12	12	12	12	12
W =	10	20	5	8	15	24	12
D =							

Note: w and d represent the weight and distance respectively on one side of the fulcrum, W and D the weight and distance on the other side of the fulcrum.



(b) Now double the weight W (make it 20 grams) and find where it must be placed to balance weight w. Write the distance in your table under "20," column (b).

(c) Make weight W only half as large as it was in the first case (make it 5 grams), adjust the balance and read the distance from the fulcrum. Write it in the table, column (c) under "5."

(d) Notice that in these first three trials, weight w and its distance from the fulcrum remained the same, and changes were made in weight W. Make at least three or four other changes in W and write the results in your table as in Table I, columns (d), (e), (f), and (g).

4. Now, as indicated in Table II, let weight w be 16 grams and its distance from the fulcrum 6 cm. Find how heavy weight W will have to be to balance at 6 cm. on the other side of the fulcrum. What weight will balance the lever 4 cm. from the fulcrum? 10 cm? Try several other distances from the fulcrum, find what weight will just balance the lever, and fill in your Table II.

TABLE II

w	16	16	16	16	16	16	16
d	6	6	6	6	6	6	6
W							
D	6	4	16				

5. Try other weights and distances as suggested in Tables III and IV and fill in similar tables of your own.

TABLE III

w	20	40	10				
d							
W	20	20	20	20	20	20	20
D	15	15	15	15	15	15	15

TABLE IV

w	18	18	18	18	18	18	18
d	5	5	5	5	5	5	5
W	15	10	30	45			
D					10	15	18

Let us now study the tables to see if there seems to be any law which expresses all relationships and which could be expressed mathematically. If there is such a law, we could use it to predict where to place any weight to balance any other weight.

Exercises

1. (a) From Table I what do you notice about the placement of equal weights on two sides of a fulcrum?

   (b) If only weight W is doubled, how does its distance from the fulcrum change?

   (c) If weight W is made half as much, how does its distance from the fulcrum change?

   (d) State a rule that seems to hold concerning w, d, W, and D.

2. Use the law that you found in Exercise 1 to predict the correct measures in the missing parts of this table.

TABLE V

w	25	35	12	45	21	15	23	11	?	14	10	100	100	100
d	3	4	5	3	5	?	4	8	7	5.5	50	50	500	5000
W	15	?	7	?	14	20	12	?	13	10	?	?	?	?
D	?	7	?	9	?	4.5	?	10	8	?	5	5	5	5

3. Go back to your equipment and check your results in several of the above exercises to see if there is a balance.

A Graph of the Experiment

In order to study a set of observations from an experiment, scientists often make a graph or drawing of them on a set of axes.

If you have not drawn such graphs, the following will give you the general idea.

Exercises

1. (a) Use graph paper and begin with two perpendicular lines called axes. The intersection of the axes is named point O. (See attached graph.)

   (b) One axis is named W and the other D, and we will locate the points corresponding to pairs of weights and distances in your Table I.

   In Table I, the first weight was 10 and the distance 12. Locate 10 on the W axis. Follow the vertical line through 10 to the point where it meets the horizontal line through 12 on the D axis. This point is called (10, 12).

   Similarly, from Table I, when W = 20 and D = 6, locate 20 on the W axis. Follow the vertical line through 20 to the point where it meets the horizontal line through 6 on the D axis. This point is called (20, 6).

   Locate the other points from your Table I. These are (10, 12), (20, 6), (5, 24), (8, 15) etc.

   Draw a smooth, freehand curve through the points you have located. This curve gives you the general picture of the relation between the weights and the distances.

If any point seems to lie rather far from a smooth curve through the other points, check your computation for that one. Maybe you've made a mistake in arithmetic.

2. By examining the graph, find answers to these questions:

   (a) What happens to the distance as the weight increases?

   (b) What happens to the weight as the distance increases?

3. Now that you have a graph of the relation between weight and distance, on a balanced seesaw, you can predict the distance corresponding to a given weight.

   In order to find the value of W when D is 24, locate 24 on the D axis and follow the horizontal line through 24 until it meets the curve. Read the value on the W scale directly beneath this point. You should have 5. (The dotted line and arrows on the graph may help you.)

   Find the values of W from the graph.

   (a) D = 30 (W = ?) (b) D = 15 (W = ?) (c) D = 9 (W = ?)

4. Tell whether or not each of the following points is on the graph.

   (a) W = 10, D = 25 (b) W = 5, D = 5 (c) W = 20, D = 15

5. Fill in the following table by selecting a value of W on the graph and following the perpendicular through that point to the curve, thus locating the corresponding value of D on the curve.

If W	6	7	8	9	16	17	21	23	25
then D	20	?	?						

Note: Mathematicians call the curve that you have drawn a <u>hyperbola</u>. It is a very interesting and useful curve.

Other examples of levers

Other forms of the type of lever which we have studied are the crowbar and the claw hammer. Make a list of other examples of this type.

There are different types of levers. You may wish to study some of these and report your findings to the class.

Archimedes once said that if he could have a long enough lever and a place to rest it, he could lift the earth. Explain.

Special projects

Many scientific facts were undiscovered for thousands of years until an alert scientist carefully set up an experiment in a manner similar to the one you have done and made discoveries on the basis of observations.

(a) For thousands of years, people assumed that if a heavy object and a light object were dropped at the same time, the heavier one would fall much faster than a light one.

Look up the story of Galileo and his experiment with falling objects and see what he discovered.

(b) From time immemorial, people watched eclipses of the sun and moon and saw the round shadow of the earth but did not discover that the earth was round. Eratosthenes, in 230 B.C., computed the distance around the world by his observations of the sun in two locations in Egypt, yet seventeen hundred years later when Columbus started on his journey, many people still believed the world was flat.

Look up in a history of mathematics book or in an encyclopedia the story of Eratosthenes and this experiment.

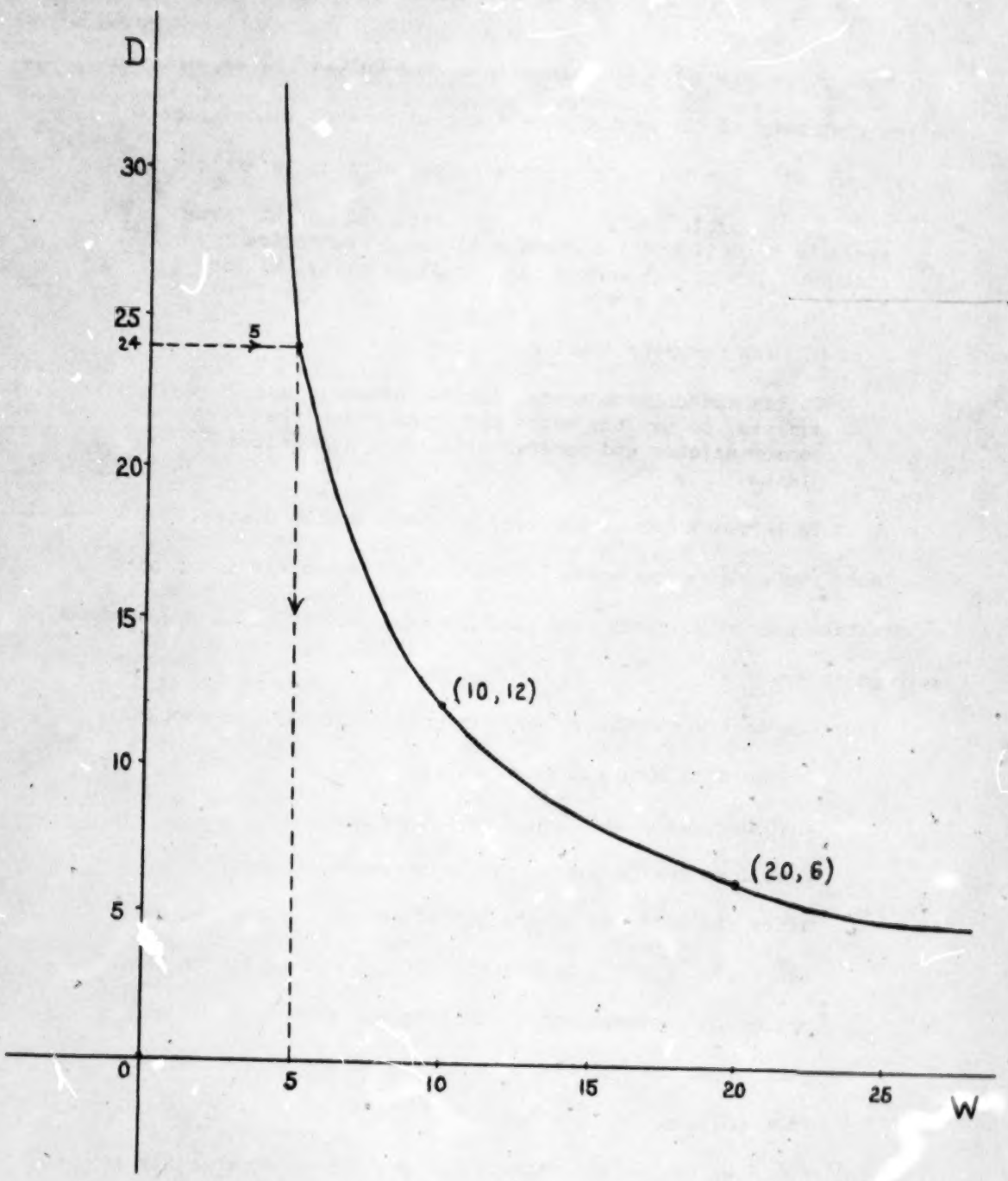
(c) People had watched pendulums swing for many centuries before Galileo did some measuring and calculating and discovered the law which gives the relation between the length of the pendulum and the time of its swing.

Look up this experiment in a history of mathematics book.

Notice that all such experiments are based on many careful measurements and observations in order to discover the scientific law, and then the law is stated in mathematical terms. Thus we see the dependence of science on mathematics.

XI - 9

The Seesaw Graph



UNIT 12

UNCLE SAM AS A STATISTICIAN

When you studied the Constitution of the United States, did you notice that many of the sections need a good deal of mathematics to carry them out? For instance, notice these sections in Article I:

> Sec. 2. Apportionment. Representatives and direct taxes shall be apportioned...according to their respective numbers. The actual enumeration shall be made...within every...term of ten years.
>
> Sec. 8. The Congress shall have power:
>
> To lay and collect taxes, duties, imposts, and excises, to pay the debts and provide for the common defense and general welfare of the United States.
>
> To borrow money on the credit of the United States.

When you examine the above provisions and speculate about the mathematics needed to carry them out, you will probably raise questions such as these:

(a) How does Uncle Sam go about counting 170 million people?
How does he sort all that information?
What use would he make of computing machines?
What does the Census Bureau do between censuses?
After the citizens of the United States are counted every ten years, and the information is used to decide on the number of representatives in Congress from each state what other uses can be made of the information that has been collected?

(b) How does Uncle Sam estimate the government expense for any year and how does he know how much to collect in taxes? How does he decide whether to lower taxes or not? Some people think that if taxes are lowered, the government will

not collect enough money to pay this year's expenses; others think that if taxes are lowered, people will buy more goods, more people will make money, and the government will make more on income taxes. Who is right? Such questions cannot be decided by personal opinions or by experiment since a trial of one or the other plan might cause a great loss to the government. So the ones who decide must study the statistics and base their decisions on them.

How does Uncle Sam decide what duties to charge on foreign imports? If duties are low, will people buy foreign products rather than American-made ones? If duties are high, how will that affect American industries like a clothing industry which manufactures its products out of imported material, or a watch assembly industry which uses foreign-made works for watches?

(c) And how does Uncle Sam decide what interest to pay on the money he borrows -- that is, on U. S. Government Bonds? If he pays a high rate of interest on his bonds, which of course are a very safe investment, will people invest their money in banks, real estate, mining companies, etc., if they pay the same rate of interest? Will such companies have to pay a higher rate of interest than Uncle Sam in order to get people to invest in them?

All such questions are answered only after a very careful study of the data or statistics that have been collected. Naturally we will not be able to answer the above questions in this unit, but we will be able to study a few ideas about statistics. In fact, the word "statistics" is derived from the word "state", for governments have always had to

keep very careful records and collect many data in order to be able to make wise decisions about questions such as those that were suggested. The method of drawing conclusions or making decisions on the basis of samples of data is called "statistical inference."

The study, sorting, and handling of statistics require mathematical skills, and the fundamentals can be studied in junior high schools.

Can you list some reasons why Uncle Sam needs to have such information as:

(a) The number of men who will reach the age of 18 next year?

(b) The total national income?

(c) The number of unemployed people?

(d) The size of the labor force?

(e) The number of people who will be 65 next year and collect Social Security payments?

(f) The amount of gasoline used in a year?

List other information which you can think of which might be needed by the federal government.

The federal government has always needed to collect statistics of many kinds. Even before the first census was taken in 1790, Alexander Hamilton started collecting statistics for the Treasury Department on foreign trade. He saw that such figures would be important for the developing of new industries in this country and for expansion of foreign trade. In the third census (1810) Thomas Jefferson ordered "an account of the several manufacturing establishments and manufacturers."

A list of the agencies which collect statistics for Uncle Sam is very extensive:

The Organization of the Federal Statistical System -- 1958

1. General Coordination: Office of Statistical Standards
   (Bureau of the Budget)

2. General-Purpose Statistical Agencies

   Statistical Branches of Agricultural Marketing Service
   (Department of Agriculture)

   Bureau of Labor Statistics
   (Department of Labor)

   Bureau of the Census
   (Department of Commerce)

   National Office of Vital Statistics
   (Department of Health, Education, Welfare)

3. Analytic and Research Agencies

   Council of Economic Advisers (Executive Office of the President)

   Office of Defense Mobilization (Executive Office of the President)

   Division of Agricultural Economics (Department of Agriculture)

   Office of Business Economics (Department of Commerce)

   Division of Research and Statistics (Federal Reserve System)

   Production Economics Research Branch (Department of Agriculture)

   Business and Defense Services Adm. (Department of Commerce)

   Bureau of Foreign Commerce (Department of Commerce)

   Bureau of Mines (Department of Interior)

4. Other Administrative, Regulatory, and Defense Agencies

A Study of Data by a Table and a Graph

So many decisions of Uncle Sam depend on the number of people in the country that we should first examine the population in various years: *

Table I
Population Facts about the United States

Year	Population in Millions	Increase in Millions per decade	Per cent of Increase per decade
1790	3.9		
1800	5.3	1.4	35.1
1810	7.2	1.9	36.4
1820	9.6	2.4	33.1
1830	12.9	3.3	33.5
1840	17.1	4.2	32.7
1850	23.2	6.1	35.9
1860	31.4	8.2	35.6
1870	39.8	8.4	26.6
1880	50.2	10.4	26.0
1890	62.9	12.7	25.5
1900	76.0	13.1	20.7
1910	92.0	16.0	21.0
1920	105.7	13.7	14.9
1930	122.8	17.1	16.1
1940	131.7	8.9	7.2
1950	150.7	19.0	14.5

* From STATISTICAL ABSTRACT OF THE UNITED STATES, 1956

Exercises

A careful study of such a statistical table will help you to answer the questions in Exercises 1 through 8.

1. Do you see any general trends in the table?
2. Do any particular figures seem surprising? Why do they seem so?
3. In which decade was the percent of increase the highest? From your study of history do you know the reason?
4. In which decade was the per cent of increase the lowest? Can you explain it by any history of that period that you have studied?
5. In which decade did the second lowest per cent of increase occur? Why?
6. When was the "Irish Famine"? How did that affect the population of the United States?
7. What was the increase in population from
   (a) 1800 to 1850? (b) 1850 to 1900? (c) 1900 to 1950?
8. What was the per cent of increase in each of the periods mentioned in Exercise 7?
9. Construct a broken line graph of the population of the U. S. A. at the end of each decade as given in the table on page 5. Then answer the following questions:
   (a) What general trend does the graph show?
   (b) In which 50-year period did the population grow the fastest?
   (c) If the population increases at the same rate from 1950 to 1960 as in the previous decade (if the graph continues as a straight line from 1940 to 1960) what will be the population in 1960?
   (d) Why does Uncle Sam need to know, at least approximately, how many people there will be in the U. S. next year? In 1960?

The following table shows the immigration to this country in recent years.

Table II
Immigration Facts about the United States

Period	Number of Immigrants	Millions
1820 - 1830	151,824	.2
1831 - 1840	599,125	.6
1841 - 1850	1,713,251	1.7
1851 - 1860	2,598,214	2.6
1861 - 1870	2,314,824	2.3
1871 - 1880	2,812,191	2.8
1881 - 1890	5,246,613	5.2
1891 - 1900	3,687,564	3.7
1901 - 1910	8,795,386	8.8
1911 - 1920	5,735,811	5.7
1921 - 1930	4,107,209	4.1
1931 - 1940	528,431	.5
1941 - 1950	1,035,039	1.0

Exercises

Use Table II to answer the following:

1. Do you see any general trends?
2. Which figures seem out of line in these trends? Explain why.
3. Examine this table along with the one on population and see if there are any similarities.
4. If you have not done Exercise 9, page 6, prepare a broken line graph showing the increase in population in each decade from 1790 to 1950. (Table I)

5. On the same page, draw a broken line graph showing immigration by decades from the above table.
6. In which periods were there increases in both populations and and immigration?
7. What was the general trend in immigration
   (a) From 1830 to 1890? Why?
   (b) From 1890 to 1900? Why?
   (c) From 1900 to 1920? Why?
   (d) From 1920 to 1940? Why?
8. In how many periods were there increases in immigration? In how many periods were there decreases?
9. There were new"quota laws" in 1923. How did they affect immigration?
10. Describe the trend in immigration if it continues in the same way as in the last 20 years. Can you make an approximate prediction for the period 1950-60? Do you see why Uncle Sam needs to know such trends in order to help get a correct estimate of population?

The Arithmetic Mean

You have now examined some of the statistics collected by Uncle Sam. You studied the tables, you constructed graphs for a closer study, and you saw how some predictions might be made on the basis of your study.

Statistics can be studied in another way, that is by using mathematics to a greater extent in studying the sets of data.

It is difficult, and often unnecessary, to have a clear mental picture of a set of data. It is helpful to try to describe the most important features by a few numbers. The most useful numbers of this kind are the arithmetic mean (sometimes called simply the mean), the median, the mode, the range, and the average deviation.

You have probably averaged several numerical grades in some classes by adding them and dividing by the number of grades. This is a useful kind of average and is called the arithmetic mean or the mean. We will use it to study a set of statistics collected by the federal government.

Table III
Number of Unemployed Persons in the U. S.

Year	(a) Number of Thousands	(b) Number of Hundred Thousands	(c) Number of Millions
1932	12,060	121	12
1934	11,340	113	11
1936	9,030	90	9
1938	10,390	104	10
1940	8,120	81	8
1942	2,660	27	3
1944	670	7	1
1946	2,270	23	2
1948	2,064	21	2
1950	3,142	31	3
1952	1,673	17	2
1954	3,230	32	3
1956	2,834	28	3

One of the first and easiest facts to find in a set of data is the range, or the difference between the highest and lowest numbers. In the previous table, we see that the range (in numbers of millions) is 12 - 1 or 11.

To find an arithmetic average or mean, find the total of the set of numbers and divide by the number of items:

The total of the numbers in column (c) is 69.

The number of items is 13.

$$\frac{\text{Total}}{\text{Number of items}} = \frac{69}{13} = 5.3 \text{ (to the nearest tenth)}$$

Notice that the mean does not tell you the number for any one year, but if the unemployment were 5.3 millions each year for 13 years, the total would be 13 x 5.3 millions or 69 millions.

Did any particular year have the "average" number of unemployed persons?

Averages are often called "measures of central tendency". Do you see why?

Exercises

1. Find the mean of the following set of test scores:

   88, 78, 92, 85, 83, 90, 88, 85, 86, 91

2. From Table III find the mean of the number of unemployed persons, in number of thousands, column (a) from 1932 through 1940.

3. From Table III find the mean of the number of unemployed persons, in number of thousands, column (a) from 1942 through 1956.

4. The following temperatures were the noon readings for a week. Find the mean noon temperature for the week:

   52°, 48°, 65°, 62°, 67°, 70°, 56°

5. Why was unemployment so high in 1932? 1934?

6. Why was it low in 1942? 1944?

7. How does the amount of unemployment affect

   (a) the number of automobiles that are sold?

   (b) the amount of unemployment compensation to be paid?

   (c) the amount of food and clothing that is bought?

The Median and the Mode

In a set of data, it is often useful to know what would be the middle number if they were arranged from smallest to largest. In such a case, half of the numbers would be larger and half would be smaller than the middle number which is called the median.

For an example, let us examine the heights of pupils in a certain class of 13 pupils.

Pupil	Inches in Height
1	53
2	57
3	61
4	62
5	51
6	61
7	54
8	59
9	61
10	60
11	65
12	52
13	53

To find the pupil in the group, so that half the group is taller and half the group is shorter, picture the pupils arranged in order of height. The measurements, arranged in order, would be:

51, 52, 53, 53, 54, 57, 59, 60, 61, 61, 61, 62, 65

By counting from the left or right, the middle of the set of 13 items would be the 7th one. In this case, it is 59, so we say that the median of this set of numbers is 59.

If the largest number in this set of figures happened to be 70 instead of 65, would that affect the median? Why?

If the lowest numbers were 41 and 42 instead of 51 and 52, would that change the median?

Let us compare the mean or arithmetic average of this set of figures with the median. The sum of the numbers is 749.

$\frac{749}{13} = 57.6$. Thus the mean is 57.6 . How does this compare with the median?

If the largest number in this set of figures were 80 instead of 65, would that affect the mean? Why?

Notice that even though the median and mean are called "averages", they may not be the same number. You will find that each is useful for particular kinds of cases.

Exercises

1. Find the median and the mean of this set of scores on an arithmetic test: 83, 81, 76, 94, 87, 71, 90, 88, 73, 95, 67, 86, 87, 89, 93
2. Make a table for the heights and weights of the pupils in your class. Calculate
   (a) the average height (mean)
   (b) the median height
   (c) the average weight (mean)
   (d) the median weight
3. Find the median temperature of the data in Exercise 4, page 10.

Grouping Data

If you were listing heights of a very large group of pupils, it might be inconvenient to list each one separately. You might group the figures in some such way as this:

Height in inches	Number of pupils
46 - 49	14
50 - 52	33
53 - 55	57
56 - 59	42
60 - 63	17
64 - 67	12

In order to find the middle pupil, find the total number and divide by 2. $14 + 33 + 57 + 42 + 17 + 12 = 175$. $\frac{175}{2} = 87\frac{1}{2}$. So the middle person will be the 88th one, counting from the top or bottom. If we count down from the top, $14 + 33 = 47$. We need 41 more to reach 88. Counting down 41 more in the group of 57 brings us to the lower part of that group. Since the 88th person is within that group, we say that the median height of the whole group of pupils is between 53 and 55 inches. Since the 88th person comes rather low in that group as we count down, we say that the median height is likely to be nearer 55 than 53.

Let's check our work and count up from the bottom to the 88th person. $12 + 17 = 29$. 42 more makes 71. So the 88th person is not in the lowest three groups. We need 17 more than 71 to make 88, so we count 17 more and that takes us into the group of 57 as we found when we counted down from the top. Again you find the 88th person in the group of 57 whose height is between 53 and 55. Thus the median height of the group is between 53 and 55 inches.

An important set of information which Uncle Sam needs each year is the data on incomes, since over 50% of the national income comes from individual income tax.

Following are the data for a recent year.

Table IV

Income	Number of Families (Thousands)
Under $1000	3,227
1000 - 2000	6,022
2000 - 3000	7,164
3000 - 4000	8,192
4000 - 5000	7,455
5000 - 6000	5,580
6000 - 7500	5,323
7500 - 10,000	3,390
10,000 - 15,000	1,899
15,000 - 20,000	523
20,000 - 25,000	274
25,000 - 50,000	336
Over 50,000	95

In a set of statistics, the value that occurs most often is called the mode.

Which income was earned by more people than any other? In other words, which is the mode of this set of figures?

You can find the mean of such a set of figures if you can find the total amount earned in each bracket. But do you know how much each of the 95,000 people earned in the "over 50,000" bracket? Might some have earned $500,000? Or $1,000,000? Or more? It is not possible to find the mean of this set of figures since we do not know how large the incomes are in the last group. Such a set of figures is called "open-ended".

It is possible, however, to find the median, or the salary bracket of the middle family. Then we will know that half of the families earned more than this amount and half earned less.

First find the total number of families and divide by two to find the middle number. Then count down from the top figure or up from the bottom figure, as taught on page 13, until you reach that number.

Exercises

1. Make a bar graph of the data in Table IV.
2. How can you identify the mode on the graph?
3. Estimate, if possible, the median salary by examining the graph carefully.
4. Find the mode in the following list of test scores: (Do not group.)

   85, 79, 82, 93, 85, 78, 82, 91, 85, 74, 93, 86, 90, 85, 78, 81, 85, 86, 94, 85, 84, 68, 83, 91, 82
5. Find the median score in the data, Exercise 4.
6. Find the median by grouping the following data on temperatures: (use intervals of 5, like 70-74, 75-79, and so on.)

   62°, 74°, 73°, 91°, 68°, 84°, 75°, 76°, 80°, 77°, 68°, 54°, 68°, 72°, 71°, 86°, 82°, 74°, 55°, 72°, 50°, 63°, 71°.

Table V
Patents Issued, 1901 - 1955

Period	Total	Number of Thousands	Total since 1900 (Thousands)
1901 - 1905	148,291	148	148
1906 - 1910	175,618	176	324
1911 - 1915	194,387	194	518
1916 - 1920	207,108	207	725
1921 - 1925	217,525	218	943
1926 - 1930	234,857	235	1178
1931 - 1935	256,219	256	1434
1936 - 1940	229,514	230	1664
1941 - 1945	184,573	185	1849
1946 - 1950	163,122	163	2012
1951 - 1955	209,215	209	2221

7. Draw a vertical bar graph of the number of thousands of patents issued by five year periods since 1900 (Table V).

8. Draw a broken line graph showing the <u>total</u> number of patents by five year periods since 1900.

9. Do you see any trends in the graphs of Exercises 7 and 8? What are they?

The Mean (or Average) Deviation

Another useful method of studying a set of numbers is to find the deviation or difference of each number from the mean.

Consider the set of numbers: 1, 2, 5, 5, 6, 7, 9.

The mean $= \frac{1 + 2 + 5 + 5 + 6 + 7 + 9}{7} = \frac{35}{7} = 5$

The deviations (differences) from the mean are:

5 - 1, 5 - 2, 5 - 5, etc., or 4, 3, 0, 0, 1, 2, 4.

The average of these deviations is:

$$\frac{4 + 3 + 0 + 0 + 1 + 2 + 4}{7} = \frac{14}{7} = 2$$

Let us see how the above measure helps to throw light on a set of data.

The total receipts of the federal government in the years 1946 - 1955 were as follows:

Year	Billions	Deviation from Mean
1946	44	- 11.5
1947	45	- 10.5
1948	46	- 9.5
1949	43	- 12.5
1950	41	- 14.5
1951	53	- 2.5
1952	68	+ 12.5
1953	73	+ 17.5
1954	73	+ 17.5
1955	69	+ 13.5
	555	

The arithmetic mean of these receipts is the total, 555, divided by 10, or 55.5.

The third column shows the difference of each year's receipts from the mean, 55.5. The - 11.5 means that 44 is 11.5 <u>below</u> the mean, for instance, and + 13.5 means that 69 is 13.5 <u>above</u> the mean. Have you seen minus signs used in a similar way to those in this table for temperature? Any other place?

Exercises

1. In which year was the deviation from the mean the greatest? Can you think of reasons for this?
2. In which year was the deviation from the mean the least?
3. Find the average deviation by finding the total of the deviations and dividing by 10. The signs before the numbers are disregarded

since we want to know the size of the deviations, no matter which direction they are from the mean.

4. Find, to the nearest tenth, the mean and average deviation of the following test scores:

   85, 82, 88, 76, 90, 84, 80, 82, 84, 83

5. Find the mean and average deviation of the test scores (same test, but in another class):

   94, 84, 68, 74, 98, 70, 96, 84, 76, 96

6. (a) Compare the means of Exercises 4 and 5.

   (b) Compare the average deviations of Exercises 4 and 5.

   (c) What information does the average deviation tell about the data which cannot be found by just using the mean?

SUMMARY OF NEW TERMS

RANGE-- the difference between the lowest and the highest numbers in the table.

ARITHMETIC MEAN-- the sum of the observations divided by the number of them.

MEDIAN-- the middle number in the set of data.

MODE-- the number that occurs most often.

MEAN DEVIATION-- the average of the deviations from the mean.

Summary Exercises

Table VI
Gross Debt of the United States for the Years 1861 to 1955
(Five-year Periods)

(a) Period	(b) Millions of dollars	(c) Billions of dollars	(d) Deviations from Mean
1861 - 65	2,678	3	
1866 - 70	2,436	2	
1871 - 75	2,156	2	
1876 - 80	2,091	2	
1881 - 85	1,579	2	
1886 - 90	1,122	1	
1891 - 95	1,097	1	
1896 - 1900	1,263	1	
1901 - 05	1,132	1	
1906 - 10	1,147	1	
1911 - 15	1,191	1	
1916 - 20	24,299	24	
1921 - 25	20,516	21	
1926 - 30	16,185	16	
1931 - 35	28,701	29	
1936 - 40	42,968	43	
1941 - 45	258,682	259	
1946 - 50	257,357	257	
1951 - 55	274,374	274	

1. (a) What is the mode of the set of figures in column (c), Table VI? In other words, which gross debt appears most often?

   (b) Does this number tell you very much about the total debt picture of the United States Treasury since 1861?

2. (a) Find the median of the set of figures in column (c) which will give you the median national debt from 1861 to 1955. This means that you will need to find the middle figure between 1 and 274. In order to do this, list the numbers from column (c) in order, and find the middle one on the list by counting up from the bottom or down from the top.

   (b) Does the median tell you much about the total debt over this period of years?

   Do you see that it is necessary to find the mean in order to take into consideration the very large numbers in the table?

3. Find the mean of the set of numbers in column (c) to the nearest whole number.

4. Find the deviations from the mean and then the average deviation to the nearest whole number.

5. Explain the large deviations both below and above the mean.

6. The following information is on salaries:

   $4,000, $6,000, $12,500, $5,000, $5,000, $7,000, $5,500, $4,500, $5,000, $6,500, $5,000

   (a) Find the mean of the data.

   (b) How many salaries are above the mean?

   (c) How many salaries are below the mean?

   (d) Does the mean seem to be a fair way to describe the "average" of this data?

   (e) Find the median of the set of data.

   (f) Does the median seem to be a fair way to describe the "average" of this data?

7. Repeat the parts of Exercise 6 using the following data on temperatures:

47°, 68°, 58°, 80°, 42°, 43°, 68°, 74°, 43°, 46°, 48°, 76°, 48°

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From the illustrations given, you have studied how Uncle Sam needs mathematics to collect and study statistics.

You have seen how mathematics is used to interpret these statistics and help the government in making decisions that affect you and me and every citizen.

The next time that you see a set of figures in the newspaper, look them over carefully, try to recognize trends, and consider whether decisions based on them, and affecting you, should be made.

UNIT XIII

CHANCE

Recently the picnic of the employees of a weather bureau was called off on account of rain. Even an expert weather forecaster cannot say for sure what tomorrow's weather will be.

If you plan a picnic you may first read the weather report for the day of the picnic. This report may read "Probable showers" or "Clear and warm." Sometimes the weatherman is correct and sometimes he is wrong. His forecast is an example of his best available estimate of what will happen. His forecast is a "Chance Statement". He, himself, does not know with complete certainty whether it is true or false. In this unit we shall study some ideas about chance statements like, "The Yankees will probably win," or "There is a 50-50 chance that a head will show if a penny is tossed and allowed to fall freely."

The table below shows a number of weather forecasts from April 1 to April 11 and, also, the actual weather.

Table

Date	Forecast	Actual weather	Truth Value of the Forecast
1.	Rain	Rain	True
2.	Light showers	Sunny	False
3.	Cloudy	Cloudy	True

Date	Forecast	Actual weather	Truth Value of the Forecast
4.	Clear	Clear	True
5.	Scattered showers	Warm and sunny	False
6.	Scattered showers	Scattered showers	True
7.	Windy and cloudy	Overcast and windy	True
8.	Thunder-showers	Thundershowers	True
9.	Clear	Cloudy and rain	False
10.	Clear	Clear	True

Each forecast is an example of a "statement". A statement is a collection of words or symbols which is either true or false (but naturally not both). In the case of the first forecast the statement was found to be true after observing the weather. Also, observations show that statements 3, 4, 6, 7, 8, and 10 were true. However, statements 2, 5, and 9 proved false since the observed weather was different from the forecast. Hereafter we shall write "true" as "T" and "false" as "F". The T's and F's are called "Truth values of the statement."

In addition to these symbols it is useful in dealing with chance statements to have a number which measures the chance that a statement is true. The weather man was correct 7 out of 10 times. Written as a ratio this is $\frac{7}{10}$. This $\frac{7}{10}$

is used as the best estimate which we can make from the given information of the measure of the chance that the next weather forecast will be true.

In this case, since we had such a small number of forecasts, it would not be fair to say that the weatherman will be correct $\frac{7}{10}$ of the time in future forecasts. To make such a conclusion requires an understanding of much more mathematics than we can study in this unit. It does, however, give us some clue as to the chance that the weatherman may be correct.

When we find the measure of the chance that statements will be true, we shall say that we are estimating the true measure from the best information we have. This might be called estimating the true value of the chance statement.

Written as a formula this is:

The measure of the chance that a statement is true =

$$\frac{\text{The number of times the statement is T}}{\text{The total number of times the statement is T or F}}$$

or

$C = \frac{T}{S}$, where T means the number of times the statement is true, S means the sum of the number of T's and F's, and C is the measure of the chance.

Exercises

1. The following statement is made:

"The stoplight at Main Street and Center Street is

green whenever I arrive at the intersection."

a. Why is this a "statement"?

b. Below is a table of the actual color when I was arrived at the intersection. Copy the table and in the column at the right, write whether the statement above is T or F. You may use "T" for true and "F" for false if you like.

Table

The light		Truth value of the Statement
1.	Green	
2.	Red	
3.	Amber	
4.	Green	
5.	Red	
6.	Amber	
7.	Red	
8.	Green	
9.	Green	
10.	Amber	
11.	Red	
12.	Green	
13.	Red	
14.	Amber	
15.	Green	
16.	Red	

c. Write a number or measure for the chance that the statement is correct.

2. Change the statement in Ex. 1 to read "Red" instead of "Green". Use the same observations as in part b and find a number or measure for the chance that the new statement is correct.

3. Given: 3 red marbles and 2 white marbles are in a box. You are to take out of the box, without looking inside it, two marbles. What is the chance that both marbles will be red? (Sometimes we merely say "What is the chance?" when we mean "What is the number which measures the chance that the statement is true?")

In this problem the chance statement is: In a drawing, two red marbles will be drawn from a box containing 3 red and 2 white marbles.

The following table lists the possible pairs which may be drawn. We assume that each red marble has the same chance or likelihood of being drawn. Let's call the three red marbles R_1, R_2, and R_3 and similarly the white marbles shall be called W_1, and W_2. Complete the table of truth values and find the number for the chance that the statement is correct. Mark as T those draws which consist of 2 red marbles.

Draws	Truth Value
1. R_1 R_2	
2. R_1 R_3	

Draws	Truth Value
3. R_1 W_1	
4. R_1 W_2	
5. R_2 W_1	
6. R_2 W_2	
7. R_2 R_3	
8. R_3 W_1	
9. R_3 W_2	
10. W_1 W_2	

4. Two black marbles and one white marble are in a box. You are to take out, without looking inside the box, two marbles. What is the chance that one marble is black and the other is white?

Suggestions: Make a table as in Ex. 3. List all possible pairs and find the truth value of each pair. You should have three pairs.

5. In Ex. 4, what is the chance that both marbles drawn are black?

6. Three hats are in a dark closet. Two belong to you and the other to your friend. Being a polite person, when your friend is ready to leave with you, you reach in the closet and draw any two hats. What is the chance that each of you gets his own hat?

7. A committee of two is selected from a group of 4 people. You are one of the 4. If the person selecting the committee

has no favorites and, therefore, selects without a plan, what is the chance that you will be on the committee?

8. In Ex. 7, your favorite friend is also one of the 4 people. What is the chance that both you and your friend will be selected?

Models for Chance Statements

You may have noticed two kinds of problems in the set of exercises above. One kind uses observations to determine whether a statement is T or F. (Exs 1, 2, and the example page 1). However, in the marble problems and the hat problem experimental observations were not used. All possible events were listed and the measure of chance was based upon these possibilities. Thus, in these cases a "model" was made to represent the problem. It is also necessary to point out that each event in a model, say the 10 pairs in Ex. 3, has the same chance of occurring. In the case of Ex. 3, each pair has a $\frac{1}{10}$ chance of occurring.

In the exercises above finding an estimation or measure of the chance that a statement is true was a matter of <u>counting</u>. First we were talking about a statement and, secondly, we were able to count all possible (or available) T's and F's. Although more advanced mathematical processes have been set up to find measures of chance where counting is impossible, counting is essential in many situations.

An easy type of model to talk about is tossing a penny or pennies. A penny has a head and a tail (let's assume here that the coin cannot stand on edge if it is tossed and allowed to fall freely on a flat surface). Also, in all problems about coins, we assume that the coin is balanced (honest). We then expect heads to appear about as many times as tails if the coin is tossed a great number of times.

Example 1: Find the measure of the chance that the statement, "If two pennies are tossed, exactly one head shows," is true.

Solution:

a. Make a table showing all possible events.

1st penny	2nd penny	Truth value
Head	Head	F
Head	Tail	T
Tail	Head	T
Tail	Tail	F

Note: In this model, each event (each possible result of the tossing of two coins), one of which is, for example, a head and a tail, has the same chance of occurring; that is, $\frac{1}{4}$.

b. There are 4 possible ways the pennies may fall. In the table above the truth value of each event has been written.

If both pennies are heads, the statement above is F; if the first penny is a head and the second is a tail, the above statement is T; and so on.

c. Count the number of times the statement is true. This number is the numerator of measure of chance. The sum of the number of T's and F's is the denominator.

The chance that exactly one head will show if two coins are tossed is $\frac{2}{4}$ or $\frac{1}{2}$ or $C = \frac{1}{2}$ in this case.

Example 2: Find the measure of the chance that the statement, "If three pennies are tossed, exactly two heads show," is true.

Solution:

a. Make a table of all possible ways the 3 coins may fall and after each event write T if the event makes the statement true and F if the event makes the statement false.

1st penny	2nd penny	3rd penny	Truth value
Head	Head	Head	F
Head	Head	Tail	T
Head	Tail	Head	
Head	Tail	Tail	
Tail	Head	Head	
Tail	Head	Tail	
Tail	Tail	Head	
Tail	Tail	Tail	

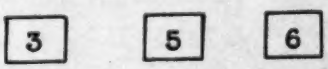
(Complete the table)

b. Count the T's for the numerator and both the T's and the F's for the denominator. Then, if 3 pennies are tossed the chance that exactly two heads show is $\frac{3}{8}$ or $C = \frac{3}{8}$.

Exercises

For each exercise find the measure of the chance statement.

1. If two pennies are tossed, exactly two heads show.
2. If two coins are tossed, at least one head shows.
3. If two coins are tossed, exactly one tail shows.
4. If three coins are tossed, exactly one head shows.
5. If three coins are tossed, exactly three heads show.
6. If three coin are tossed, at least one head shows.
7. In Example 1 on page 8, the table shows four events.
   a. For each event, find the measure of the chance that it occurs.
   b. Find the sum of the four measures of chance.
8. Do the same for example 2.
9. If four coins are tossed, what is the chance of exactly 3 heads showing? Of exactly 1 tail showing? Of exactly two heads showing? Of exactly four heads showing?
10*. Three cards are numbered like the ones below.



Without looking at the numbers you are to draw any two cards. What is the chance that the sum of the two numbers is odd?

11*. You are to be placed in a line with two girls (or boys), one of whom is your favorite. What chance is there that you will stand next to your favorite? In such a problem we

assume that you are not placed according to any plan (including your own). If you crowd in next to your favorite, chance would not play a role.

Decisions and Chance

Suppose a person must make a choice between two possibilities. Should he just choose one by guessing or is there a better method for him to try? A person may make a wiser decision by observing and comparing statements about the chance of the two events in question.

Example: Which of the two following statements is more likely to be true?

A. If 3 pennies are tossed, exactly two heads appear.

B. If 2 pennies are tossed, exactly one head appears.

Solution: A

The measure of the chance of A = $\frac{3}{8}$

The measure of the chance of B = $\frac{1}{2}$

Conclusion: Since $\frac{1}{2}$ is greater than $\frac{3}{8}$, statement B is more likely to be true.

In order to solve a problem of this kind, find for each statement the measure of the chance that it will be true and select the statement corresponding to the larger measure.

Exercises

For each exercise find a measure of the chance statement for each event and choose, if possible, the event most likely to happen.

1. Statement A: If three coins are tossed, exactly 2 tails show.

   Statement B: If two coins are tossed, exactly 1 head shows.

2. A: If 3 coins are tossed, at least 2 heads show.

   B: If 2 coins are tossed, exactly 1 tail shows.

3. A: If 4 coins are tossed, exactly 3 heads show.

   B: If 3 coins are tossed, exactly 2 heads show.

4. A: It rains on Friday the 13th.

   B: The sun shines on Friday the 13th.

The following table shows the weather on 20 Friday the 13ths. From the table decide which is more likely to happen over a great number of Friday, the 13th's.

	Truth Value of Statement A	Truth Value of Statement B
1. Heavy rain		
2. Light rain		
3. Sunny		
4. Usually sunny, no rain.		
5. Sunny		
6. Scattered showers		
7. Showers		
8. Sunny		
9. Sunny		

	Truth Value of Statement A	Truth Value of Statement B
10. Sunny		
11. Cloudy, no rain		
12. Partly cloudy		
13. Cloudy with some showers		
14. Showers		
15. Sunny		
16. Sunny		
17. Hot and sunny		
18. Sunny		
19. Cloudy and some showers		
20. Sunny		

Experiments

1. a. If one penny is tossed what is the chance that a head shows?

   b. How many heads would you expect to get if the penny is tossed 50 times?

   c. Toss a penny 50 times and compare the observed results with the measure found in part b. You are extremely lucky if they agree exactly.

2. Keep a record (as in the Example on page 1) of weather forecasts of a morning paper and an evening paper. Find chance statements for each paper and compare their measures.

3. In Ex. 3, page 5, the chance was estimated by making a "model" of the possible events.

The solution of this problem could be approached by carrying on an experiment. For example, repeat the drawing of a pair 50 times (after each drawing replace the 2 marbles that you have drawn). Each draw then makes the statement T or F. Apply our method for calculating a measure of a chance statement.

Compare this measure to the measure obtained from the model. It is not very likely that the two measures each found in a different way, will be exactly alike. In a further study of chance you will study more about how close you could expect the two measures to be.

4. Set up an experiment to answer the question: What chance is there that exactly two heads will show if three coins are tossed?

5. Set up an experiment to answer the question. What chance is there that exactly 5 heads will show if ten coins are tossed?

6. a. Mark off on heavy cardboard parallel lines, two inches apart.

   b. Make ten sticks from wooden matches (or tooth picks) 1 inch long.

   c. Hold the ten sticks about 8 or 9 inches above the cardboard and drop them.

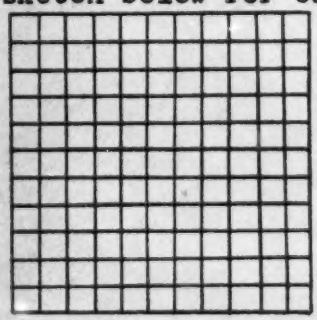
d. Count the number of sticks which touch or cross a line of the cardboard.

e. Repeat the experiment 50 times, each time counting and recording the number of sticks which touch or cross a line.

f. Find the average number of sticks that touch or cross a line out of the 10 that are dropped.

7. You are to decide whether or not commercial fertilizer helps seeds germinate and grow rapidly.

Set up the following experiment to help you make a decision: Bring two boxes about a foot square and 6 inches deep. Fill each with soil from the same area. With the help of your science teacher make sure that the moisture in the two boxes of soil is about equal. Then add commercial fertilizer to one of the boxes. Make a grid with strings like the sketch below for each box.



Two sets of 10 strings. The strings of each set are 1 inch apart.

At each intersection, plant corn seeds. There will be 100 seeds planted in each box. Add the same amount of water to each box and place both boxes in the same part of the room. Consult your science teacher in regard to the amount of water to be added. The seeds should all be planted the

same depth in the soil.

a. Decide how you would use chance to help decide whether or not fertilizer helped the seeds germinate.

b. Decide how you would further your experiment to decide whether or not the plants grew more rapidly under the conditions of commercial fertilization.

8. Some members of the class may carry on the experiment described in Ex. 7 but first they should test the soil to determine the best kind and amount of fertilizer to use. The science teacher may be able to help with this, the county agent may have soil testing kits to lend, sell, or give away, or, finally, there may be a nearby soil testing station to which a sample may be taken.

9. Repeat the type of experiment described in Ex. 7 but put fertilizer in both boxes. However, add twice as much in one and decide if the extra fertilizer produced better results.